An analysis of the MPR selection in OLSR and consequences

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Analysis of the Multi-Point Relays selection in OLSR and Implications

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***Abstract*— Optimized Link State Routing protocol (OLSR) is a promising routing protocol for multi-hop wireless ad-hoc networks, recently standardized by the IETF as a RFC. OLSR uses intensively the concept of Multi-Point Relays (MPR) to minimize the overhead of routing messages and limit the harmful effects of broadcasting in such networks. In this article, we are interested in the performance evaluations of the Multi-Point Relay selection. We analyze the mean number of selected MPR in the network and their spatial distribution with a theoretical approach and simulations. Then, we discuss the implications of these results on the efficiency of a broadcasting in OLSR and on the reliability of OLSR when links between nodes may fail.**

`Keywords`:ad hoc, multi-hop, wireless, OLSR, MPR, performances, Palm.

I. Introduction

Due to the emergence of wireless local area network technologies such as 802.11 [3], hiperlan [4] or bluetooth [5], the use of mobile wireless networks is growing fast. With these technologies, new challenges arise such as connecting wireless nodes without any infrastructure. In order to connect nodes which are not in each other's radio range, packets need to be relayed by intermediate nodes. Such networks thus require forwarding capabilities on each node and a routing protocol to find the available path to any destination. Routing in such a wireless environment is very different from classical routing in wired networks. Indeed, nodes are mobile by essence and may vanish or appear due to the wireless nature of the physical layer. The topology is thus in constant evolution. However, routing advertisements are expensive in resources since a node spends energy while transmitting as well as receiving and each message sent by a node is received systematically by all its neighbors. Therefore, the number of broadcast advertisements must be limited, but also the number of nodes which propagate them through the network, in order to maximize the network lifetime. Consequently, an accurate routing protocol needs to be distributed, must guarantee a low level of traffic control overhead but should be able to quickly take into account link failures due for instance to node movements. The Internet Engineering Task Force (IETF) addresses the design of such protocols in its MANET[1] (Mobile Ad-hoc Networks) working group.

One of the recent proactive standardized protocols is OLSR (Optimized Link State Routing Algorithm) presented in [1], [2]. Proactive, or table-driven, routing protocols deeply rely on network broadcasting features and proactive routing protocols aim to reduce the impact of message flooding in order to limit their control overhead and reach scalability. In OLSR, only a subset of preselected nodes called MPR (Multi-Point Relays) are used to perform topological advertisements. At the same time, control messages (containing *e.g.* routing information) are broadcast and forwarded only by MPR. Thus, the number of emitter nodes is reduced, overhead and useless receptions of messages on nodes are thus minimized and the well known storm problem [6] due to broadcasting is avoided.

In this article, we are interested in the performances of the MPR selection. We analyze the mean number of selected MPR in the network and their spatial distribution using a theoretical approach and by simulations. We then show that the algorithm used for selecting the MPR is efficient for certain quantities as the mean number of redundant packets

[1] http://www.ietf.org/html.charters/manet-charter.html

received by a node during a broadcasting task and that the different proposed variants of the algorithm always lead to very close performances (as at least 75% of the selected MPR are the same nodes whatever the selection algorithm). We also discuss the implication of the different analytical results on the reliability of the protocol.

The remaining of the paper is as follows. In Section II, we briefly detail the OLSR protocol and the MPR selection algorithm. In Section III, we give results about probabilities and mean quantities relative to the MPR selection algorithm. We then discuss about the implication of these results on the performances of OLSR in Section V. Numerical results and simulations are presented in Section IV. We lastly conclude and discuss of future works in Section VI.

II. OLSR

OLSR is a proactive routing protocol for Mobile Ad-hoc Networks (MANET), *i.e.*, it permanently maintains and updates a network topology view on each node in order to provide a route as soon as needed. It uses the concept of Multi-Point Relays to minimize the overhead of control traffic and to provide shortest path routes (in number of hops) for all destinations in the network. Each node chooses a subset of nodes in its neighborhood as its MPR. A MPR set is thus relative to each node. Each node also keeps the list of its neighbors which have selected itself as a MPR. This list is called the MPR-selector list. It is obtained from HELLO packets which are periodically sent between neighbors. In order to build the database to route the packets, all the MPR broadcast their MPR-selectors in the network. The shortest path to all possible destinations is then computed from these lists, a path between two nodes being a sequence of MPR.

Since only MPR are authorized to send their MPR-selectors, the control traffic is drastically reduced compared to the classical link-state algorithms. When receiving a broadcast message M from a node u, a node v forwards it if and only if it is the first time v receives M and if node u is in node v's MPR-selectors list. This technique allows to reduce the number of transmitters of broadcast messages. We detail the algorithm which allows a node to select its MPR within its neighborhood. It consists of choosing nodes in such a way that the whole 2-neighborhood is covered. In this way, MPR are selected in order to reach the 2-neighborhood in two hops, the k-neighborhood of the source node is reached within k hops. Paths are thus the shortest expected paths in number of hops.

A. MPR selection

As the optimal MPR selection is NP-complet [10], we give here the Simple Greedy MPR Heuristic which is the one currently used in the OLSR implementation.

For a node u, let $N(u)$ be the neighborhood of u. $N(u)$ is the set of nodes which are in u's range and share a bidirectional link with u. (If $v \in N(u)$ then $u \in N(v)$.) We denote by $N_2(u)$ the 2-neighborhood of u, *i.e*, the set of nodes which are neighbors of at least one node of $N(u)$ but which do not belong to $N(u)$. ($N_2(u) = \{v \text{ s.t. } \exists\, w \in N(u) \,|\, v \in N(w) \setminus \{u\} \cup \ N(u)\}$). A message sent by node u and relayed by a node $v \in N(u)$ reaches a node $w \in N_2(u) \cap N(v)$ in two hops.

For a node $v \in N(u)$, let $d_u^+(v)$ be the number of nodes of $N_2(u)$ which are in $N(v)$:

$$d_u^+(v) = |N_2(u) \cap N(v)|$$

This quantity represents the number of nodes of $N_2(u)$ that node u can reach in two hops via node v.

For a node $v \in N_2(u)$, let $d_u^-(v)$ be the number of nodes of $N(u)$ which are in $N(v)$:

$$d_u^-(v) = |N(u) \cap N(v)|$$

This quantity represents the number of nodes in $N(u)$ which allow to connect nodes u and v in two hops.

Node u selects in $N(u)$, a set of nodes which covers $N_2(u)$ integrally. We define as $MPR(u)$ this set of MPR selected by u. In other words, $MPR(u)$ is such that:

$$u \cup N_2(u) \subset \bigcup_{v \in MPR(u)} N(v)$$

This algorithm is run on every node and selects the MPR in two steps. It supposes that each node knows its 1-neighbors and its 2-neighbors. In the remaining of the paper, we denote by MPR_1 the nodes selected during the first step.

The algorithm is the following:

Algorithm 1 Simple Greedy MPR Heuristic

For all node $u \in V$
  $N'(u) = N(u)$ and $N'_2(u) = N_2(u)$.
  ▷ *First step*
  For all node $v \in N(u)$
    if $(\exists w \in N(v) \cap N_2(u) \,|\, d^-_u(w) = 1)$ **then**
      Select v as $MPR(u)$.
      ▷ *Select as $MPR(u)$, the nodes of $N(u)$ covering "isolated" nodes,* i.e. *for which there is a neighbor in $N_2(u)$ which has v as single parent in $N(u)$.*
      Remove v from $N'(u)$ and remove $N(v) \cap N_2(u)$ from $N'_2(u)$.
    end
  ▷ *Second step*
  while $(N'_2(u) \neq \emptyset)$
    For all node $v \in N'(u)$
      if $(d^+_u(v) = max_{w \in N'(u)} d^+_u(w))$ **then**
        Select v as $MPR(u)$.
        ▷ *Select as $MPR(u)$ the node v which cover the maximal number of nodes in $N_2(u)$.*
        Remove v from $N'(u)$ and remove $N(v) \cap N_2(u)$ from $N'_2(u)$.
      end

The first step selects as MPR the nodes which cover "isolated nodes of $N_2(u)$". We denote a node $v \in N_2(u)$ by "isolated node in $N_2(u)$" if there is only one node w in $N(u) \cap N(v)$ which allows to connect v and u in two hops (v is such that $d^-(v) = 1$).

To better understand this algorithm, let's run it on the green node u on Figure 1. The isolated points of node u appear in red and MPR_1 in blue. For instance, node t is an isolated node via node h as only node h in $N(u)$ allows to connect t and u in two hops. Node h is thus elected during the first step: node h is a MPR_1.

So, during the first step, node u will select the red nodes as MPR_1 and nodes $k, j, t, s, r, q, o, m, l$ in $N_2(u)$ will be covered by these MPR_1.

Then, node u goes to step two. It only considers nodes of $N_2(u)$ which are not already covered (nodes p and n) and nodes in N_1 not selected as MPR_1 (nodes b, f, e and d). That means that it only keeps the view of the topology illustrated by Figure 1(b). It will first select the one which has the highest degree on Graph 1(b). Node e covers nodes n and p whereas nodes f and d cover only one node in $N_2(u)$ (resp. nodes p and n).

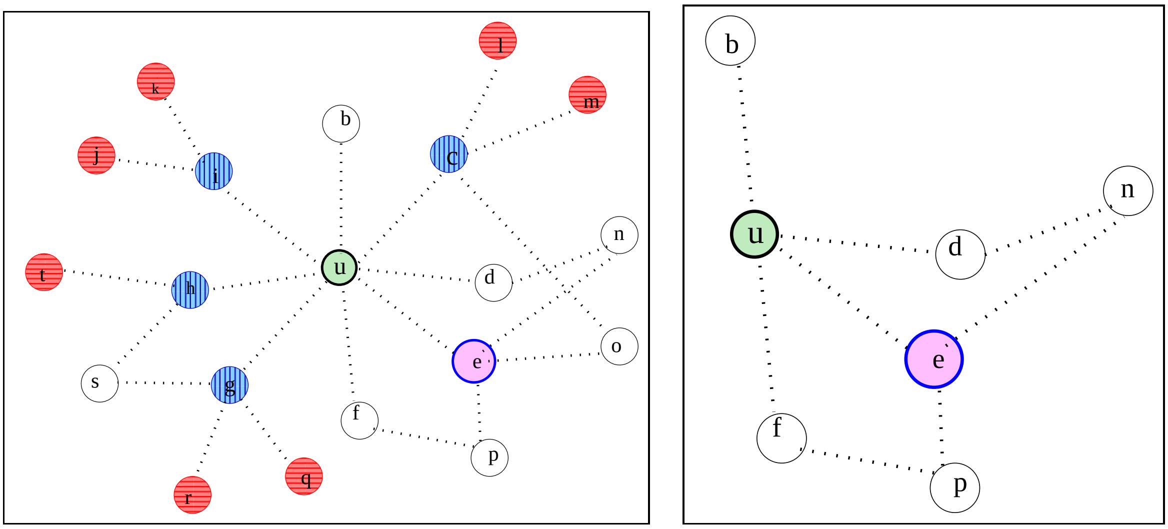
From here, all nodes of $N_2(u)$ are covered by the selected MPR, the algorithm stops. We have: $MPR(u) = \{c, e, i, h, g\}$.

From here, it is easy to see that nodes of $N(u)$ which cover "isolated nodes" must be included into the set of MPR if we want to cover the whole 2-neighborhood whatever the selection process. Thus, we can not skip or "compress" the first step of the algorithm in the MPR selection. Moreover, this step must be run first in order to minimize the number of MPR.

Therefore, only the second step of the algorithm can be improved in order to find the minimum number of MPR.

B. Related works

Most of the literature about the performances of OLSR deals with the efficiency of the OLSR routing protocol itself or the different flooding techniques using MPR ([7], [8], [9], [10]). As the goal is to minimize the number of transmitters when propagating some information (*e.g.* routing information) in the network in order to maximize its lifetime, the number of selected MPR per node has to be as low as possible. Therefore, alternative algorithms to the classical MPR algorithm described above as [13], [12] are given in order to reduce the number of collisions, minimize the overlap between MPR or maximize the global bandwidth. But, all results for the proposed algorithms are quite similar, particularly for the mean number of MPR per node. Therefore, in order to understand this phenomenon, we wished to analyze this selection more in details as only few papers have studied the different algorithm performances of the MPR selection. An analysis of the MPR selection on the line is given in [13]. Other analytical results in random graphs



(a) Global topology - Isolated points of node u appear in horizontal red, MPR_1 of u appear in vertical blue.

(b) Topology considered by node u at the second step

Fig. 1. Illustration of the MPR selection algorithm.

and random unit graphs are also given in [8]. For instance, a rough upper bound on the size of a node's MPR set is given in a random unit graph. Other interesting results are presented in [12].

III. Analysis

We are interested in the properties of the MPR set of a typical node. Therefore, we do not consider the whole network but only a "typical point" located at the origin of the plane and its 1 and 2-neighborhood. Our model is similar to the classical unit random graph used to model ad-hoc networks. This family of models is not completely realistic since it omits interference between the nodes. More realistic models have been proposed, for instance in [14] where the authors present an accurate model for a CDMA network. However, we have chosen a more general model since we do not make any assumptions about the wireless technology used by the nodes.

Let $B(x,R)$ denote a ball of radius R centered in x. Let be a Poisson point process on $B(0,2R)$ of intensity $\lambda > 0$. We add a point 0 at the origin for which we study the MPR selection algorithm (Palm distribution). The intensity λ of such a process represents the mean number of points of the process by surface unit. We assume that there is a bidirectional link between two nodes if and only if $d(u,v) \leq R$ where $d(u,v)$ is the Euclidean distance between u and v and $R \in I\!R^{+*}$ a constant.

The neighborhood of the point 0 is thus constituted of the points of the Poisson point process which are in $B(0,R)$. We use the notation already defined in Section II: N (resp. N_2) is the 1-neighborhood (resp. the 2-neighborhood) of the point 0.

A. General results

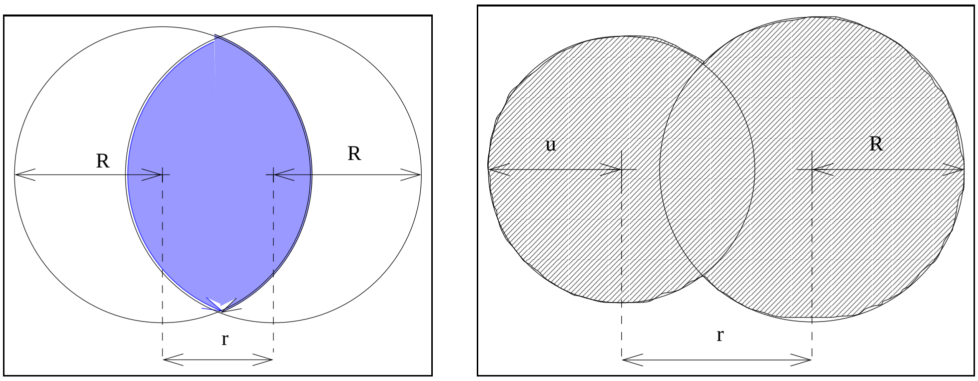
Let's note $A(r)$ the area of the intersection of two balls of radius R where the distance between the centers of the balls is r, illustrated on Figure 2(a):

$$A(r) = 2R^2 \arccos\left(\frac{r}{2R}\right) - r\sqrt{R^2 - \frac{r^2}{4}}$$

and $A_1(u,r,R)$ the area of the union of two discs of radius R and u where the centers of the two balls are distant from r, illustrated on Figure 2(b):

$$A_1(u,r,R) = rR\sqrt{1 - \left(\frac{R^2 - u^2 + r^2}{2Rr}\right)^2}$$
$$-R^2 \arccos\frac{u^2 - R^2 - r^2}{2Rr} - u^2 \arccos\frac{R^2 - u^2 - r^2}{2ur}$$

The next proposition gives several general results as the mean values of the quantities d_0^+ and d_0^- as well as the mean size of the 1 and 2-neighborhood of a node when considering a Poisson point process distribution.



(a) $A(r)$ is the blue area: intersection area of two balls of radius R.

(b) $A_1(u,r,R)$ is the lined area : union area of two discs of radius R and u.

Fig. 2. Illustration of areas $A(r)$ and $A_1(u,r,R)$.

***Proposition** 1:* Let u be a point uniformly distributed in $B(0,R)$. u is thus such that $u \in N$.

The mean number of node u's neighbors lying in $B(0,2R)/B(0,R)$ is given by:

$$\begin{aligned} \mathbb{E}\left[d_0^+(u)\right] &= \frac{\lambda}{\pi R^2}\int_0^{2\pi}\int_0^R (\pi R^2 - A(r))rdrd\theta \\ &= \lambda R^2 \frac{3\sqrt{3}}{4} \end{aligned}$$

The idea is to count the number of the process points lying in the intersection of $B(u,R)$ and $B(0,2R)/B(0,R)$.

Let v be a point uniformly distributed in $B(0,2R)\backslash B(0,R)$. The mean number of node v's neighbors lying in $B(0,R)$ is given by:

$$\mathbb{E}\left[d_0^-(v)\right] = \lambda\frac{2}{3R^2}\int_R^{2R} A(r)rdr = \lambda R^2\frac{\sqrt{3}}{4}$$

The idea is here to count the number of process points in the intersection of $B(v,R)$ and $B(0,R)$. Node v may lie in $B(0,2R)\backslash B(0,R)$ without belonging to N_2 if $N(v)\cap N = \emptyset$. So, to obtain the quantity above for nodes in N_2 we have to condition it by the probability that $v \in N_2$. We obtain:

$$\mathbb{E}\left[d_0^-(v)|v\in N_2\right] = \frac{\mathbb{E}\left[d_0^-(v)\right]}{\mathbb{P}(d_0^-(v)>0)}$$

with

$$\mathbb{P}\left(d_0^-(v)>0\right) = 1 - \frac{2}{3R^2}\int_R^{2R} exp\{-\lambda A(r)\}rdr$$

This last equation gives the probability that a node in $B(0,2R)/B(0,R)$ has at least one neighbor in $B(0,R)$ which makes it a 2-neighbor of node 0.

The mean number of nodes in N is given by:

$$\mathbb{E}\left[|N|\right] = \lambda\pi R^2$$

The mean number of nodes in N_2 is given by:

$$\begin{aligned} \mathbb{E}\left[|N_2|\right] &= 3\lambda\pi R^2\mathbb{P}\left(d_0^-(v)>0\right) \\ &= 3\lambda\pi R^2 \times \\ &\quad \left(1 - \frac{2}{3R^2}\int_R^{2R} exp\{-\lambda A(r)\}rdr\right) \end{aligned}$$

All these quantities can be computed in the same way. We use the following properties of a Poisson point process: conditioned by the number of points in $B(0,R)$ (resp. in $B(0,2R)\backslash B(0,R)$), the points are independently and uniformly distributed in $B(0,R)$ (resp. in $B(0,2R)\backslash B(0,R)$) and are independent of the points of $B(0,2R)\backslash B(0,R)$ (resp. $B(0,R)$). For instance, we are able to find the distribution of $d_0^+(u)$ the number of points of N_2 covered by a point u of $B(0,R)$: it is a discrete Poisson law of parameter $\lambda\nu(B(u,R)\backslash B(0,R))$ (ν is the Lebesgues measure in $I\!R^2$) with u uniformly distributed in $B(0,R)$.

B. Analysis of the first step of the MPR selection

In this section, we compute several quantities relative to the first step of the algorithm. We use

MPR_1 to denote the set of points of N which are selected as MPR during the first step of the algorithm. In the next proposition, we give the mean number of points $v \in N_2$ such that $d_0^-(v) = 1$. These points are the isolated points. The points of N, neighbors of these isolated points, necessarily belong to MPR_1 as they are the only way to reach them from node 0 in a minimum number of hops. However, this quantity does not give the size of MPR_1, since several isolated points can be reached by the same MPR_1 point. For instance, on Figure 1(a), we have four MPR_1 nodes but seven "isolated points". The MPR_1 i covers two isolated points: nodes j and k. By definition, the "isolated points" of node u are the nodes $v \in N_2(u)$ such that $d_0^-(v) = 1$.

***Proposition** 2:* Let v be a point uniformly distributed in $B(0, 2R)\backslash B(0, R)$ and D the set of points v such that $d_0^-(v) = 1$.

$$\mathbb{P}\left(d_0^-(v) = 1\right) = \frac{2}{3R^2}\int_R^{2R} \lambda A(r) exp\{-\lambda A(r)\} r dr$$

As in Proposition 1, we only consider nodes v such that $v \in N_2$:

$$\mathbb{P}\left(d_0^-(v) = 1 | v \in N_2\right) = \frac{\mathbb{P}\left(d_0^-(v) = 1\right)}{\mathbb{P}\left(d_0^-(v) > 0\right)}$$

The mean number of "isolated points" is then deduced and given by:

$$\mathbb{E}\left[|D|\right] = 2\pi\lambda^2 \int_R^{2R} A(r) exp\{-\lambda A(r)\} r dr$$

In the next proposition we give a lower bound and an upper bound of the mean size of the MPR_1.

***Proposition** 3:* Let u be a point uniformly distributed in $B(0, R)$.

$$\mathbb{P}\left(u \in MPR_1\right) \geq \frac{2}{R^2}\mathbb{P}\left(d_0^+(u) > 0\right)$$
$$\times \int_0^R \int_R^{R+r} f(x, r, R)$$
$$\times \exp\left\{-\lambda\left(2\pi R^2 - A_1(R, x, R)\right)\right\} r dx dr$$

with $f(x, r, R)$ being the probabilistic distribution function:

$$f(x, r, R) = -\frac{\lambda}{1-\exp\{-\lambda(A_1(R,r,R)-\pi R^2)\}} \times$$
$$\left[\frac{\partial}{\partial x}A_1(x, r, R) - 2\pi x\right] \times$$
$$\exp\left\{-\lambda\left(A_1(x, r, R) - \pi x^2\right)\right\}$$

The next formula gives the mean number of MPR_1. It is the direct consequence of the formula above:

$$\mathbb{E}\left[|MPR_1|\right] \geq 2\lambda\pi\mathbb{P}\left(d_0^+(u) > 0\right) \times$$
$$\int_0^R \int_R^{R+r} f(x, r, R) \times$$
$$\exp\left\{-\lambda\left(2\pi R^2 - A_1(R, x, R)\right)\right\} r dx dr$$

Moreover, since there is at least one isolated point by point of MPR_1, the mean number of isolated points offers an upper bound:

$$\mathbb{E}\left[|MPR_1|\right] \leq \mathbb{E}\left[|D|\right]$$

Proof: We just give here a sketch of the proof. We obtain a bound on the probability that a point in N belongs to MPR_1. A sufficient condition that $u \in MPR_1$ is that the farthest point of $N(u)$ from 0, denoted w, is such that $d_0^-(w) = 1$. Given the distance of u from 0 (expressed by r in the formula), we calculate the probabilistic distribution function of the distance of w and deduce the density function of the distance between w and 0 (denoted $f(x, r, R)$ in the formula). Given the distance between w and u, we are able to compute the probability that $d_0^-(w) = 1$.

This bound is very accurate since, in most cases, the isolated points are the farthest points from node 0. ■

We are also interested in the spatial distribution of the MPR_1 points. For u, a neighbor of 0 at distance r ($u \in N$ such that $d(0, u) = r$) $r \leq R$, we give a lower bound and an upper bound on the probability that u belongs to MPR_1. For it, we consider a point u at distance r ($r \leq R$) from the origin. We fix the two points 0 and u and we distribute the Poisson point process in $B(0, 2R)$ independently of these two points. From it, we analyze the probability that this node u be a MPR_1 in function of r.

Proposition 4: Let u be a point at distance r ($r \leq R$) from the origin. We fix the two points 0 and u and we distribute the Poisson point process in $B(0, 2R)$ independently of these two points.

$$\mathbb{P}\left(u \in MPR_1\right) \geq \left(1 - \exp\left\{-\lambda(\pi R^2 - A(r))\right\}\right) \times \int_R^{R+r} f(v, r, R) \exp\left\{-\lambda(2\pi R^2 - A_1(R, v, R))\right\} dv$$

$$\mathbb{P}\left(u \in MPR_1\right) \leq 1 - \left(1 - \exp\left\{-\lambda \frac{A(R+r)}{2}\right\}\right)^2$$

Proof: The lower bound is obtained by the same way as the bound in Proposition 3 but given the distance between the origin and its neighbor u. We point out that according to the n-fold Palm distribution for Poisson point process, the considered process corresponds to the conditional distribution of the process given the locations of the two fixed points (c.f. [15], page 124, for more details). The upper bound is obtained as follows. If there is a point in the two semi-intersections as illustrated in Figure 6(a), almost all the neighbor nodes of u which belong to N_2 are covered by these nodes and therefore are not isolated. For the points which are not covered by one of these points we may easily show that the same bound holds. This gives a lower bound on the probability that u does not belong to MPR_1 from which we deduce the upper bound on the probability that u belongs to MPR_1. ∎

IV. Numerical results and simulations

In simulations, the nodes of the network are represented by a Poisson point process in $B(0, 2)$ ($R = 1$) of intensity $\lambda > 0$. We add a point at 0. We study for this point the number of MPR selected at each step of the MPR selection and we show that the analytical results are very close to the simulations' ones. In Figures 3(a), 3(b), 4(a) and 4(b), we have represented samples of the model for different values of $\lambda\pi$. $\lambda\pi$ represents the mean number of neighbors of a node in the network for such a Poisson process. The point at the origin for which we compute the MPR is the black point in the middle of the figures. The points in the central circle represent the set N (the neighbors of 0). The larger points of this set represent the MPR_1 points (points selected as MPR during the first step of the algorithm). The points outside the circle are the points of N_2 (the 2-neighborhood of 0) and the blue points are the points of N_2 which are covered by the MPR_1 points.

We note that in all four cases, almost the entire 2-neighborhood of 0 is covered by the MPR_1 set. The addition of one MPR might suffice to cover the rest of N_2. We have shown in the previous section that there is an appreciable number of isolated points giving rise to a certain number of MPR_1 points. These MPR_1 points seem to be distributed very close to the boundary of $B(0, R)$ and regularly scattered on it (which confirms the results of the Proposition 4). Therefore, they cover a very large part of N_2.

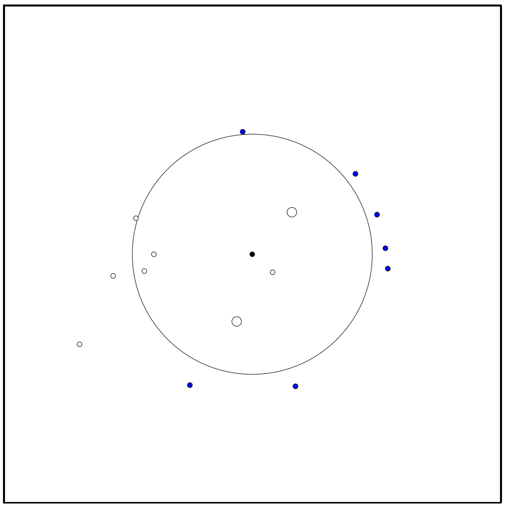
Figure 5(a) shows the mean number of MPR and MPR_1 obtained by simulation. We observe that approximately 75% of the MPR are in MPR_1 which confirms that the MPR_1 cover almost the whole 2-neighborood. In Figure 5(b), we have plotted the mean number of MPR_1 obtained by simulation and the analytic lower bound. As explained before, the lower bound is very close to the mean size of the set MPR_1.

The lower and upper bounds on the probability that a point belongs to MPR_1 described in Proposition 4 allow us to show that the MPR_1 points are very close to the boundary. In Figure 6(b), these bounds are plotted when the distance between 0 and its neighbors varies from 0.2 to 0.999 and with $\lambda = 15$. These curves incontestably show that MPR_1 points are distributed closely to the boundary of $B(0, 1)$. We point out that these results depend on λ: as λ increases, the distance between MPR_1 points and 0 increases too.

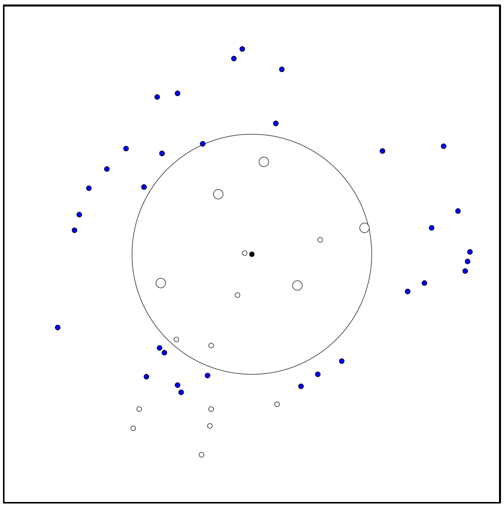
V. Consequences

A. About the distribution of the MPR

When a message is broadcast, when node u emits, the nodes in $MPR(u)$ forward the message, so, the common neighbors to u and its MPR will receive

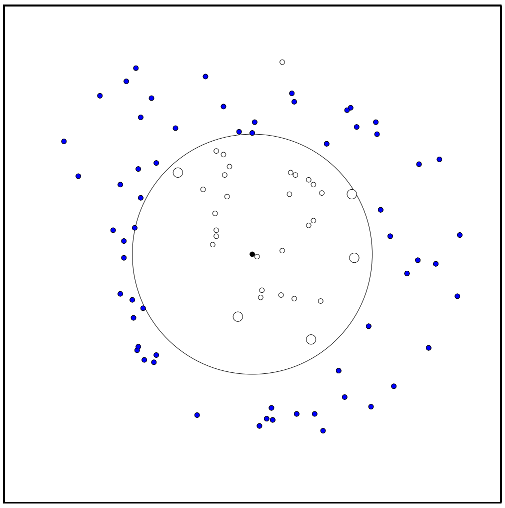


(a) $\lambda\pi = 6$

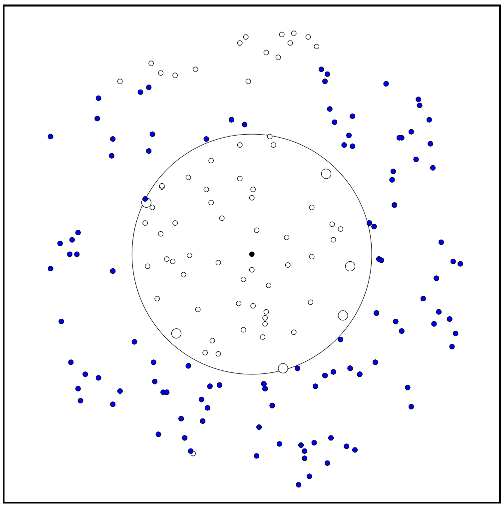


(b) $\lambda\pi = 15$

Fig. 3. MPR selection with $\lambda\pi = 6$ and $\lambda\pi = 15$.



(a) $\lambda\pi = 30$



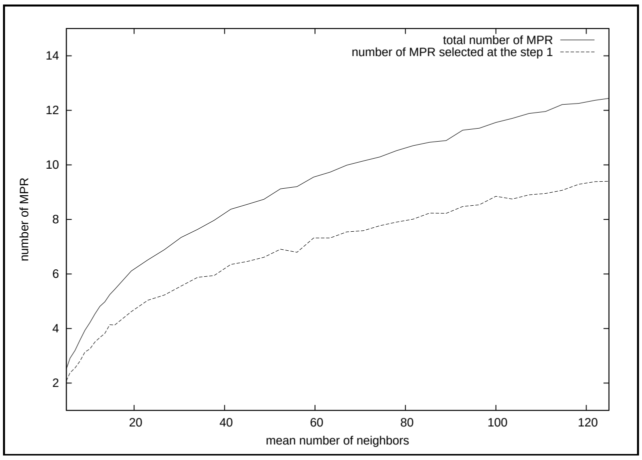
(b) $\lambda\pi = 45$

Fig. 4. MPR selection with $\lambda\pi = 30$ and $\lambda\pi = 45$.

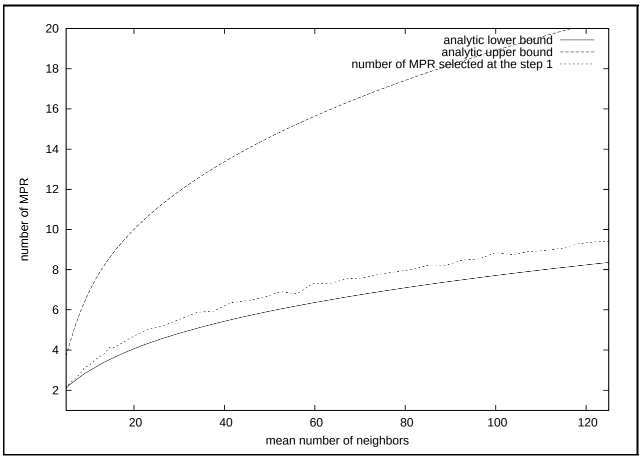
several copies of the same message and will spend energy uselessly.

Yet, as shown in Section IV, most of MPR_1 (and thus most of $MPR(u)$) are distributed very closely to the boundaries of the radio range of u. That means that the number of common neighbors of u and its $MPR(u)$ is minimized, as Figure 2(a) shows. Indeed, r the greater, the blue area the smaller, the number of common neighbors being directly proportional to this area. So, as described in [6], for messages broadcast over the network, part of the redundancy perceived by nodes is linked to the size of the intersection between the MPR radio areas. Since the distance between a point and its MPR is great, these intersections are minimal, thus minimizing the redundancy.

The easiest way to broadcast a message over a network is the blind flooding, *i.e.*, each node re-emits the message upon first reception of it. To illustrate the number of receptions spared by the MPR, we computed by simulation the mean number of receptions per node of a message broadcasted in the network by a randomly source. We used a simulator we developed. The geometric approach used in the analysis allows to model the spatial organization of networks. The nodes are randomly

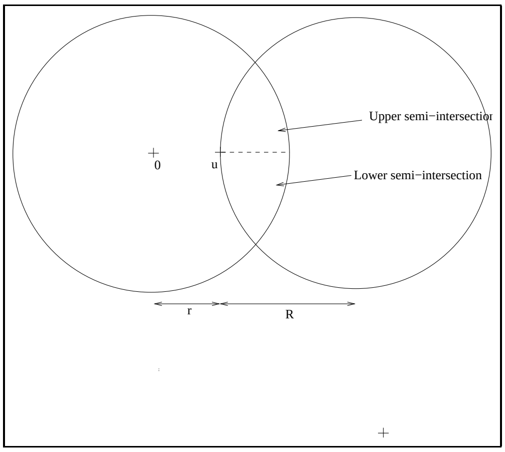


(a) Mean number of MPR and MPR_1 obtained by simulation when $\lambda\pi$ varies.

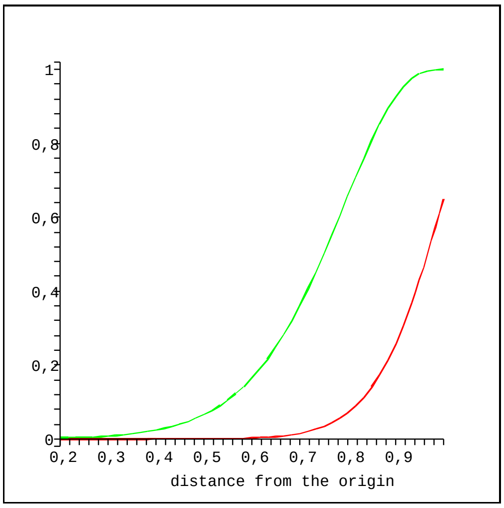


(b) Comparison of the number of MPR_1 obtained with simulations and the analytical bounds when $\lambda\pi$ varies.

Fig. 5. Mean number of MPR and MPR_1 obtained by simulation and comparison with the analytic bounds.



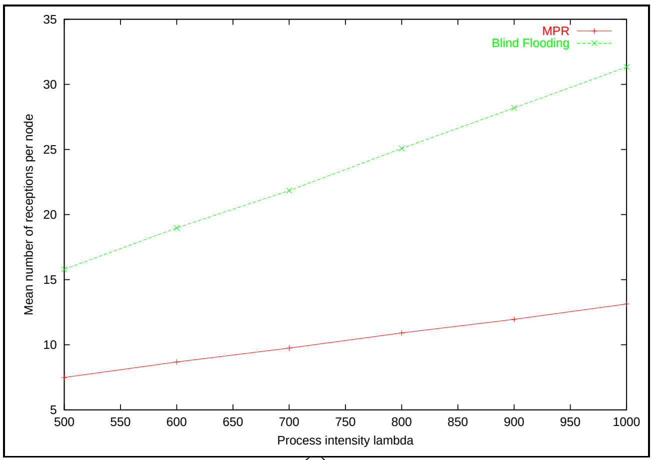
(a) The two semi-intersections used in the proof of Proposition 4.



(b) Lower and upper bounds on the probability of belonging to MPR_1 *w.r.t.* the distance from the origin.

Fig. 6. Semi-intersections used in the proof of Proposition 4 and the bounds on the probability of belonging to MPR_1 *w.r.t.* the distance from 0.

deployed using a Poisson process in a $(1 + 2R) \times (1 + 2R)$ square with various levels of intensity λ (and thus various numbers of nodes). Statistics are measured on nodes situated on a centered square of 1×1 in order to avoid the edge effects. The communication range R is set to 0.1 in all tests. Two nodes (x, y) are connected if and only if $d(x, y) \leq R$ where d is the Euclidean distance. Each statistic is the average over 1000 simulations and we fix a minimum radius and/or number of nodes such that the network is connected. Figure 7 compares the results obtained by both metrics for a process intensity $\lambda = 1000$. For the blind flooding, the number of receptions per node corresponds to the mean number of neighbors (as every node forwards the message once). With OLSR, thanks to the use of the MPR, when a broadcasting task is performed in the whole network, approximately 40% of the nodes participate to the diffusion. It is drastically less than the blind flooding and it is a priori sufficiently high to be robust *i.e.* with this rate of useless receptions (redundancy), a low level of link failure should not lead to the loss of packets during the broadcasting operation.



(a)

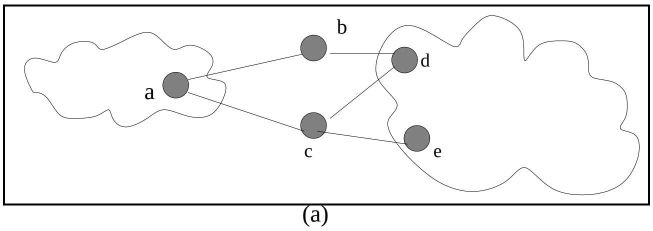
Fig. 7. Mean number of receptions per node of a message initiated at a random source and broadcast through the whole network for $\lambda = 1000$.

B. About the MPR_1

As we saw in previous sections, the goal of introducing the MPR is to minimize the number of re-transmitters. Thus, the number of MPR elected per node should be as low as possible. We presented here the Greedy heuristic of selection of MPR. It is the original one. As we mentioned in Section II-B, some works have been lead in order to try to enhance this algorithm and elect less MPR per node. But, only the second step of the Greedy algorithm may be improved as the first one is mandatory to cover the whole 2-neighborhood of a node and can not be reduced. And, as we could see in Sections III and IV, the first step leads to the election of more than 75% of the MPR. This means that the improvements can only concern less than 25% of the MPR and thus can not be significant. This explains the fact that all works searching to improve the MPR selection lead to similar results and minor improvements.

Unfortunately, this feature also underlines a robustness problem. Indeed, if 75% of node u's MPR cover at least one isolated node in $N_2(u)$ and if some $MPR(u)$ fail, there is a great probability that at least one node v in $N_2(u)$ does not receive messages from u. Of course, this does not mean that v will not receive the message at all as it could receive it from another path but, this path would be longer and the routing information won't be optimal anymore. Because of it, a node v such that $v \in MPR(u)$ and $v \notin MPR(w)$ may first receive a broadcast message by w so it would not forward it even if it then receives it by u. This can isolate some parts of the network during the broadcasting task as illustrated by Figure 8. Clouds represent parts of the network connected by nodes b and c. When the node a chooses its MPR, as the node e is an isolated point, it has to elect node c as one of its MPR. It will not elect node b as the node covered by b (node d) is already covered by c. Let's suppose that node c fails and a diffusion is performed by node a before it re-computes its MPR. The network is still connected nevertheless, as node b is not a MPR of node a, it will not forward the message and thus a whole part of the network is not informed by the broadcasting task and not only the isolated point. We have seen in the previous section, that the MPR selection involves a high rate of reception redundancy (between 40 and 50%). But, in situations where a maximum of redundancy is required (as shown in Figure 8), the algorithm offers a poor number of alternative paths for the broadcasting, leading to a low reliability.

In order to measure this robustness problem, we simulate a broadcasting task for comparing the blind flooding and the MPR heuristic. We apply a failure probability over links and measure the proportion of nodes still receiving the broadcast message. As in blind flooding, every node transmit upon first re-



(a)

Fig. 8. Example.

ception of the message, nodes which do not receive the message only if the network is disconnected. Figure 9 shows the results for a process intensity $\lambda = 1000$.

As in the blind flooding, every node retransmit the message, if some nodes do not receive it, that means that the network is disconnected. We can see that this happens when 85% or more of links are down. However, every node does not receive the message with the MPR heuristic when only 45% of links are down whereas the network is still connected.

This failure model may seem not very realist as links can fail because of congestion and, as the blind flooding induces more messages than the MPR protocol, more links will fail. Nevertheless, we use the results of the blind flooding in this situation to give us an information on the network connectivity. However, failures of a MPR may also be due to the mobility of the nodes. Indeed, if a MPR moves, it may leave the radio scope of the node for which it is a MPR or does not cover the same set of nodes in the 2-neighborhood anymore. This is particularly true with MPR_1, isolated points are close to the edge of the radio scope of the MPR_1, therefore small moves of the MPR_1 quickly involve a disconnection with their isolated nodes. These moves are not instantly taken into account by the protocol and thus may introduce an unexpected behavior during a broadcasting operation.

VI. Conclusion

In this article, we have computed several quantities relative to the MPR selection algorithm in OLSR. We have shown that approximately 75% of the MPR are chosen during the first step of the algorithm. Since this step always is necessary for the MPR set to cover the whole 2-neighborhood, variants of the algorithm used in OLSR, trying to minimize the number of selected MPR, lead to similar performances. We have also highlighted the fact that these MPR are distributed close to the radio range boundaries, limiting the overlap between MPR. This feature also underlines a robustness problem. This robustness problem is intented to be analyzed with other robustness models. A deeper study about the influences of isolated points on the reliability of OLSR will be lead in future works. These results have been presented for a particular model using Poisson point process. Other models, more realistic, which take into account the properties of the radio layer could be considered in future works. Results obtained here could be compared to simulations considering CDMA network or 802.11 network.

References

[1] Optimized Link State Routing Protocol T. Clausen, P. Jacquet, A. Laouiti, P. Muhlethaler, A. Qayyum et L. Viennot, IEEE INMIC Pakistan 2001.

[2] Optimized Link State Routing Protocol (OLSR), Clausen, T. and P. Jacquet, Eds., RFC 3626, October 2003.

[3] standards.ieee.org/getieee802/802.11.html

[4] HIPERLAN functionnal specifications, ETS 300-652, 1996.

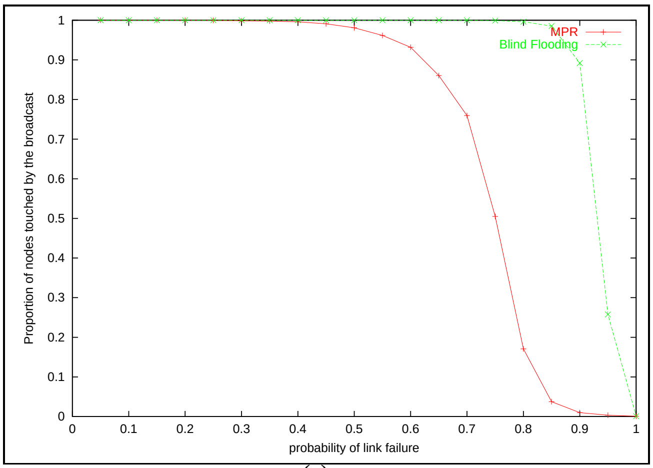
[5] www.bluetooth.org/specifications.htm

[6] The broadcast storm problem in a mobile ad hoc network. S.-Y. Ni, Y.-C. Tseng, Y.-S. Chen, and J.-P. Sheu. In Proc. of the 5th annual ACM/IEEE Int. Conference on Mobile computing and networking, pages 151–162. ACM Press, 1999.

[7] Simulation Results of the OLSR Routing Protocol for Wireless Network A. Laouiti, P. Muhlethaler, A. Najid, E. Plakoo, Med-Hoc-Net, Sardegna, Italy 2002.

[8] Performance of multipoint relaying in ad hoc mobile routing protocols P. Jacquet, A. Laouiti, P. Minet, L. Viennot, Networking 2002, Pise(Italy)2002.

[9] The Optimized Link State Routing Protocol, Evaluation through Experiments and Simulation T.H. Clausen, G. Hansen, L. Christensen and G. Behrmann, IEEE Symposium on "Wireless Personal Mobile Communications", September 2001.



(a)

Fig. 9. Proportion of nodes still receiving a broadcast message when applying a failure probability on links, for $\lambda = 1000$.

[10] Performance Analysis of OLSR Multipoint Relay Flooding in Two Ad Hoc Wireless Network Models P.Jacquet, A. Laouiti, P. Minet and L. Viennot, Research Report-4260, INRIA, September 2001, RSRCP journal special issue on Mobility and Internet.

[11] Analysis of mobile Ad hoc Protocols in Random Graph Models. P. Jacquet and A. Laouiti, Research Report RR-3835, INRIA, December 1999.

[12] Performance Evaluation of Approximation Algorithms for Multipoint Relay Selection Bernard Mans and Nirisha Shrestha, Med-Hoc-Net'04, Bodrum, Turkey, June 27-30, 2004.

[13] Flooding techniques in mobile Ad-Hoc networks. E. Baccelli and P. Jacquet. Research Report RR-5002, INRIA, 2003.

[14] Connectivity in ad-hoc and hybrid networks. O. Dousse, P. Thiran, and M. Hasler. In Proc. IEEE Infocom, New York, NY, USA, June 2002.

[15] Stochastic geometry and its applications. D. Stoyan, W.S. Kendall and J. Mecke, Ed John Wiley and Sons.